A Simulation Model for Space-Time-Frequency Variant Fading Channels

Henrik Schulze
University of Applied Sciences South Westphalia
Lindenstr. 53, D-59872 Meschede, Germany
Email: schulze@fh-swf.de, Tel.: (+49) 291 9910 300

Abstract—We apply the Monte-Carlo simulation model to a frequency selective fading channel including space-variance. This model is suited to simulate the performance of a multi-antenna system for a moving receiver (or transmitter). It takes into account that the time-variance of the channel due to this motion has its origin in the space-variance. We emphasize that for such a channel, time and frequency correlations cannot be separated.

I. INTRODUCTION

A multi-antenna system mounted on a moving vehicle must be described by a channel with space-variant and frequency selective fading. If the time-variance of the channel is caused solely by the motion of the receiver (or transmitter), it is closely related the space-variance: The vehicle simply moves through a spatial interference pattern. A physically reasonable approach must take this into account rather separating the spatial correlations from the time correlations. This is in contrast to the case of external sources of time variance that are due to moving scatterers between transmitter and receiver.

The statistical description of time-variant and frequency-selective fading channels has been developed in the classical paper of Bello [1]. To use this model for computer simulations, this statistical model of the physical world has to be approximated by a statistical simulation model that can be implemented by a software program. One such approach to implement Bello’s statistical model is the Monte-Carlo simulation model developed by Schulze [2] (see also [3] for a more detailed justification of the underlying assumptions) and Hoeher [4]. In the present paper, we extend this model to include the space-variance of the channel. It is based on the model for the statistical space-time-frequency variant model that has been worked out by Fleury [5].

II. THE PHYSICAL MODEL

We consider a scenario of a static receiver located at a variable position \( \mathbf{x} \), see Figure 1. For simplicity, we consider only two-dimensional wave propagation parallel to the ground. We assume that all (static) scatterers are sufficiently far away so that we can deal with plane wave propagation and \( \alpha \), the angle of incidence, does not depend on \( \mathbf{x} \). We use the complex baseband description for the signals. We assume that the signal bandwidth is small compared to the center frequency \( f_0 \) so that we can deal with a constant wavelength \( \lambda_0 \). The vector \( \mathbf{l}_\alpha = \lambda_0^{-1} (\cos \alpha, \sin \alpha)^T \) points into the direction of the incoming wave.

\( I \lambda = \lambda_0^{-1} (\cos \alpha, \sin \alpha)^T \) points into the direction of the incoming wave.

A. General System Theory

Let the complex baseband transmit signal be given by \( s(t) \). The corresponding receive signal at location \( \mathbf{x} \) is given by (see Equation (2) in [5])

\[
    r(t, \mathbf{x}) = \int_{-\infty}^{\infty} d\tau \int_{-\pi}^{\pi} d\alpha e^{j2\pi f\tau} x g(\tau, \alpha) s(t - \tau),
\]

where \( g(\tau, \alpha) \) is a family of random variables that describes the contribution of the fading channel corresponding to the angle of arrival (AoA) \( \alpha \) and delay \( \tau \). This random linear system can be characterized by the space-dependent transfer function

\[
    H(f, \mathbf{x}) = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f\tau} \int_{-\pi}^{\pi} d\alpha e^{j2\pi k_{\mathbf{x}} \cdot \alpha} g(\tau, \alpha)
\]

which is a stochastic process in two variables, \( f \) and \( \mathbf{x} \). For the family of random variables \( g(\tau, \alpha) \), we adopt the uncorrelated scattering (US) assumption [1]:

\[
    E \{ g(\tau, \alpha) g^*(\tau', \alpha') \} = \delta(\tau - \tau') \delta(\alpha - \alpha') S(\tau, \alpha)
\]

In that equation, \( S(\tau, \alpha) \) is the (delay-angle dependent) scattering function. It can be interpreted as an angular and delay density of the channel transfer power. The condition (3) has the important consequence that the correlation between \( H(f_1, x_1) \) and \( H(f_2, x_2) \) depends only on \( d = x_1 - x_2 \) and \( \Delta f = f_1 - f_2 \). It can easily be shown that the auto-correlation function (ACF) \( \mathcal{R}(\Delta f, d) = E \{ H(f_1, x) H^*(f_2, x) \} \) can be expressed by the scattering function according to

\[
    \mathcal{R}(\Delta f, d) = \int_{-\infty}^{\infty} d\tau e^{-j2\pi \Delta f \tau} \int_{-\pi}^{\pi} d\alpha e^{j2\pi k_{\mathbf{x}} \cdot \alpha} S(\tau, \alpha).
\]

978-1-4244-6072-4/10/$26.00 ©2010 IEEE 311
We now introduce time-variance and consider a vehicle that is moving with constant velocity \( v \). This means that we have to replace the location \( x_i \) of antenna number \( i \) by \( x_i(t) = x_i + vt \), where \( x_i \) is the relative position of the antenna on the vehicle. The time-variant transfer function for this scenario is given by

\[
H(f, x_i + vt) = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f\tau} \int_{-\pi}^{\pi} d\alpha e^{j2\pi f\alpha} v(t) g(\tau, \alpha).
\]

The quantity

\[
l_\alpha \cdot v = \nu_{max} \cos \alpha
\]

can directly be interpreted as the Doppler shift corresponding to the angle \( \alpha \). Here we have defined the maximal Doppler shift

\[
\nu_{max} = \frac{v}{\lambda_0}
\]

and assumed a vehicle moving into x-direction, i.e. \( v = (v, 0)^T \).

The correlation \( E \left[ H(f_i, x_i + vt_i) H^*(f_k, x_k + vt_k) \right] \) between the transfer function at different frequencies, different antenna locations, and different time instants with indices \( i \) and \( k \) is given by

\[
\mathcal{R}(\Delta f, d + v\Delta t) = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f\tau} \int_{-\pi}^{\pi} d\alpha e^{j2\pi f\alpha} d\theta e^{j2\pi f\alpha} v\Delta t S(\tau, \alpha).
\]

Here we have used the abbreviations \( \Delta f = f_i - f_k \), \( d = x_i - x_k \), and \( \Delta t = t_i - t_k \). It is obvious from this equation that the temporal and spatial correlations are directly connected and cannot be treated separately.

For the analytical calculation of pair-error probabilities for an OFDM system with non-ideal time and frequency interleaving as described in [3], these correlations must be utilized. For coded BPSK in a Rayleigh fading channel, as an example, the pair error probability of an error event with Hamming distance \( L \) is given by

\[
P_L = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} \frac{1}{1 + \sin^2 \theta} \frac{E_s}{N_0} d\theta.
\]

In that equation, \( E_s \) is the BPSK symbol energy, \( N_0 \) is the noise density, and the \( \lambda_i \) are the eigenvalues of the \( L \times L \) correlation matrix \( \mathbf{R} \) with elements

\[
\rho_{ik} = \mathcal{R}(f_i - f_k, x_i - x_k + v(t_i - t_k))
\]

that are given by Equation (7). The relevant BPSK symbol error events are received at frequencies \( f_i \), time instants \( t_i \), and relative antenna positions \( x_i \). For more details, see Sec. 4.4.3 in [3].

### B. Simplifications of the Model

For most theoretical derivations or software channel simulations, it is necessary or convenient to introduce some additional assumptions about the scattering function \( S(\tau, \alpha) \). Even though it is physically unreasonable to separate time and frequency selectivity, another factorization assumption may be reasonable. In case that no evidence for this assumption would lead to false or distorted results, a preferred simplification is to split the scattering function into two factors that depend only on one variable:

\[
S(\tau, \alpha) = S_{\text{delay}}(\tau) S_{\text{angle}}(\alpha)
\]

The first factor is called the power delay spectrum (PDS), and the second part is called the power azimuth spectrum (PAS). This factorization assumption is surely a simplification of the reality because it puts constraints on the shape of the possible topographical environments, but in most cases it is not expected that this simplification would affect the results too much.

The assumption (10) leads to the factorization of the ACF given by Equation (4):

\[
\mathcal{R}(f, x) = \mathcal{R}_{\text{freq}}(f) \mathcal{R}_{\text{space}}(x)
\]

This means that the correlations are assumed to be separable into a frequency factor given by

\[
\mathcal{R}_{\text{freq}}(f) = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f\tau} S_{\text{delay}}(\tau),
\]

and a spatial factor given by

\[
\mathcal{R}_{\text{space}}(x) = \int_{-\pi}^{\pi} d\alpha e^{j2\pi f\alpha} S_{\text{angle}}(\alpha).
\]

Remember that the time correlations are included in the spatial correlations by replacing \( x \) by \( x + vt \) for a moving vehicle.

A popular model for the PDS is the exponential

\[
S_{\text{delay}}(\tau) = e^{-\tau/\tau_m} \epsilon(\tau)
\]

which is characterized by only one parameter \( \tau_m > 0 \), the delay spread. Here we have written \( \epsilon(\tau) \) for the unit step function. The corresponding frequency ACF is given by

\[
\mathcal{R}_{\text{freq}}(f) = \frac{1}{1 + j2\pi f \tau_m}.
\]

One can extend the model (14) to more general scenarios by superposition of several such exponentials with different delays and different delay spreads.

For the PAS, the isotropic scattering model

\[
S_{\text{angle}}(\alpha) = \frac{1}{2\pi}
\]

is the most popular one. The corresponding spatial ACF is given by

\[
\mathcal{R}_{\text{space}}(x) = J_0 \left( \frac{2\pi |x|}{\lambda_0} \right),
\]

where \( J_0(x) \) is the (ordinary) Bessel function of order zero. However, other distributions with an anisotropic shape should also be considered for multiple antenna systems because

312
anisotropy has a significant influence on the spatial correlations and, thus, on the error probabilities given by Equation (8). Anisotropic scattering can be handled by expanding the PAS into a Fourier series

\[ S_{\text{angle}}(\alpha) = \sum_{n=-\infty}^{\infty} c_n e^{j n \alpha} \]  

with Fourier coefficients \(c_n \in \mathbb{C}\). One can insert this into Equation (13) and use the integral representation of the Bessel functions \(J_n(x)\) to get (see \([6], [7]\))

\[ R_{\text{space}}(x) = \sum_{n=-\infty}^{\infty} 2\pi f_j c_n e^{j n \beta} J_n \left( 2\pi \frac{|x|}{\lambda} \right). \]  

Inside a circle of a given radius, the Bessel functions decrease very fast with increasing order. Exact bounds can be found in \([7]\). If only a few coefficients \(c_n\) with small absolute values of \(n\) are relevant, one can approximately express \(R_{\text{space}}(x)\) by a finite series with a leading term at \(n = 0\) that is proportional to the expression (17) for isotropic scattering and some correction terms proportional to \(e^{j n \beta} J_n \left( 2\pi \frac{|x|}{\lambda} \right)\) (for \(n \neq 0\)) that characterize the anisotropy.

This method can be applied if the Fourier coefficients \(c_n\) of the PAS \(S_{\text{angle}}(\alpha)\) are known. These coefficients can be obtained either by channel measurements or by an analytical model for the PAS. An example for such an analytical model will be presented in the numerical example given below. We point out that this method allows the numerical calculation of pair error probabilities according to Equation (8) for a wide class of anisotropic PAS models.

C. The Doppler Spectrum for one Antenna

Consider a one-antenna receiver moving with constant velocity \(v\) in x-direction. Then the antenna location at time \(t\) is given by \(x = vt\) with \(v = (v; 0)^T\). We insert this into Equation (13), substitute

\[ \nu(\alpha) = \nu_{\text{max}} \cos \alpha \leftrightarrow \alpha(\nu) = \arccos \left( \frac{\nu}{\nu_{\text{max}}} \right), \]

and get

\[ R_{\text{space}}(vt) = \int_{-\nu_{\text{max}}}^{\nu_{\text{max}}} d\alpha e^{j 2\pi \nu t} S_{\text{Doppler}}(\nu) \]

with the Doppler spectrum defined by

\[ S_{\text{Doppler}}(\nu) = \frac{S_{\text{angle}}(\nu(\nu)) + S_{\text{angle}}(-\alpha(\nu))}{\nu_{\text{max}}^2 - \nu^2}. \]

If we insert the isotropic PAS of Equation (16), we obtain the famous Jakes Doppler spectrum \([8]\). Further refinements due to anisotropy can be included by using the Fourier expansion of Equation (18).

The Doppler spectrum is a useful quantity to describe the time correlations for a moving one-antenna receiver. For more antennas, the PAS is needed to describe the space-time correlations. From Equation (22), it is obvious that the Doppler spectrum is uniquely determined by the PAS, but the converse is not true. Thus, for multi-antenna systems, one must describe the joint space-time variance by the PAS and one should better not even to use the term ‘Doppler spectrum’.

III. The Simulation Model

In order to describe Rayleigh fading, we assume that \(H(f, x)\) in Equation (2) is a zero-mean complex Gaussian process in the variables \(f\) and \(x\). Following the method described in \([2], [3], [4]\) for a time variant transfer function, we model this Gaussian process by the sum of a large number \(N\) of independent harmonic functions

\[ H_{\text{model}}(f, x) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j \theta_n} e^{-j 2\pi f n t} e^{j 2\pi n x}. \]  

with \(I_n = \lambda_0^{-1} \begin{pmatrix} \cos \alpha_n \sin \alpha_n \end{pmatrix}^T\). Due to the Central Limit Theorem, this sum approaches a Gaussian process in the limit \(N \rightarrow \infty\). The random tuples \((\theta_n, \tau_n, \alpha_n)\) must have the same statistics for each \(n\), and they are statistically independent for different values of \(n\). The random phase \(\theta_n\) is assumed to be distributed uniformly over the unit circle. The random angle of incident, \(\alpha_n\), and the random delay, \(\tau_n\), have a joint probability density function \(p(\tau, \alpha)\). By the same method as applied in \([2], [3], [4]\), one can show that the model process (38) approaches the statistical properties of \(H(f, x)\) in the limit \(N \rightarrow \infty\) if one chooses a random generator according to \(p(\tau, \alpha) = S(\tau, \alpha)\).

For multiple receive antennas mounted on a vehicle moving into x-direction with velocity \(v\), the time-variant model transfer function

\[ H_i(f, t) = H(f, x_i + vt) \]

we have dropped the index ‘model’ to simplify the notation) for antenna number \(i\) at the relative location \(x_i = (x_i, y_i)^T\) is given by

\[ H_i(f, t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j \zeta_{i,n}} e^{j 2\pi \nu t} e^{-j 2\pi \tau_n}. \]

Here we have introduced the space-dependent phase

\[ \zeta_{i,n} = \theta_n + \frac{2\pi}{\lambda_0} (x_i \cos \alpha_n + y_i \sin \alpha_n). \]

and the Doppler frequency

\[ \nu_n = \frac{v}{\lambda_0} \cos \alpha_n. \]

To simulate the performance of a transmission system, the model has to be stated in the time domain: For a complex baseband transmit signal \(s(t)\), the (noise-free) receive signal \(r_i(t)\) at antenna number \(i\) is given by

\[ r_i(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j \zeta_{i,n}} e^{j 2\pi \nu t} s(t - \tau_n). \]

For a line-of-sight path and describe Rician fading, a constant additive term has to be included in Equation (25).

IV. A Numerical Example

We seek for an analytical model for \(S_{\text{angle}}(\alpha)\) that contains a certain degree of anisotropy which is controlled by one parameter. Several such models for \(S_{\text{angle}}(\alpha)\) are proposed in the literature such as the von Mises distribution, (wrapped)
The parameter $\alpha$ is suitable both for theoretical investigations and for computer simulations of the channel. The parameter $\alpha$ is necessary for the opening of the angle of arrival for the PAS. For $\alpha \to 0$, the PAS becomes more and more isotropic, and for $\alpha \to 1$ it approaches a $\delta$ peak. The upper part of Figure 2 shows this PAS for some values of $\alpha$. The Fourier coefficients of this distribution can be expressed analytically [10] by

$$c_\alpha = \frac{a^{[n]}}{2\pi}. \quad (30)$$

We now consider the scenario of Figure 3 with two antennas separated by the distance vector $d_{12}$. The angle $\beta_{12}$ is the angle between the vector $d_{12}$ and the direction of the maximal power, i.e. the maximum of $S_{\text{angle}}(\alpha)$. The lower part of Figure 2 shows the absolute value of corresponding correlation coefficient $|\rho_{12}|$ in dependence of the distance $d_{12} = \lambda/2$ (upper figure).

Figure 2. The PAS $S_{\text{angle}}(\alpha)$ for the wrapped Cauchy distribution (upper figure) and the corresponding correlation coefficient $|\rho_{12}|$ in dependence of the angle $\beta_{12}$ for fixed distance $d_{12} = \lambda/2$ (lower figure).

Assuming that the power is normalized to one, $S_{\text{angle}}(\alpha)$ is a probability density function (PDF) on the unit circle. A wide class of such functions are given by 'wrapped' distributions. They are obtained by taking the modulo-$2\pi$-value of a random variable that is defined on the real axis.

The PDF of such a random variable defined on the circle is the 'aliased' version of the PDF of a random variable on the real line. The Fourier series of such an aliased function can thus be obtained from the samples of the Fourier transform of the original function.

We propose to use the wrapped Cauchy distribution with density function [10]

$$p(\alpha; a) = \frac{1}{2\pi} \left( \frac{1}{1 + a^2 - 2a \cos \alpha} \right) \quad (29)$$

as a model for $S_{\text{angle}}(\alpha)$ because it has a simple Fourier expansion and a simple random number generator. This makes it suitable both for theoretical investigations and for computer simulations of the channel. The parameter $\alpha$ with $0 < \alpha < 1$ controls the opening of the angle of arrival for the PAS. For $\alpha \to 0$, the PAS becomes more and more isotropic, and for $\alpha \to 1$ it approaches a $\delta$ peak. The upper part of Figure 2 shows this PAS for some values of $\alpha$. The Fourier coefficients of this distribution can be expressed analytically [10] by

$$c_\alpha = \frac{a^{[n]}}{2\pi}. \quad (30)$$

We now consider the scenario of Figure 3 with two antennas separated by the distance vector $d_{12}$. The angle $\beta_{12}$ is the angle between the vector $d_{12}$ and the direction of the maximal power, i.e. the maximum of $S_{\text{angle}}(\alpha)$. The lower part of Figure 2 shows the absolute value of corresponding correlation coefficient $|\rho_{12}|$ in dependence of the distance $d_{12} = \lambda/2$ between antenna 1 and 2 as a function of the angle $\beta_{12}$. One can see from Figure 2 that the correlation is maximal if $d_{12}$ is parallel to that direction ($\beta_{12} = 0$ or $\beta_{12} = \pi$), and it is minimal if $d_{12}$ is perpendicular ($\beta_{12} = \pm \pi/2$). For these extrema, the dependence of $|\rho_{12}|$ on the distance is plotted in Figure 4. The correlation decays very slowly if both antennas are in line with the angle of maximal power (upper figure). This is the worst case that must be taken into account. If e.g. a correlation coefficient $|\rho_{12}| \lesssim 0.4$ is requested, an antenna separation of several wavelengths is necessary for $a \gtrsim 0.6$. Figure 5 shows the ACF as a function of $(x, y)$ in the pseudo-color representation for $a = 0.6$.

![Figure 3. Definition of the angle $\beta_{12}$.](image)

A random number generator for the wrapped Cauchy distribution can be obtained from the random number generator for the Cauchy distribution as follows. Let $\phi$ be a random angle that is uniformly distributed between $-\pi/2$ and $\pi/2$. Then $x = b \tan \phi$ is a random variable that is distributed according to the Cauchy PDF

$$p_{\text{Cauchy}}(x) = \frac{1}{b} \frac{1}{1 + (x/b)^2}. \quad (32)$$

The parameter $b > 0$ controls the shape of the curve. It is related to the parameter $a$ in Equation (29) by

$$a = e^{-b}. \quad (33)$$

An good overview can be found in [10].
The wrapped Cauchy random variable is then obtained from the Cauchy random variable by taking the modulo $2\pi$ value.

Figure 6 shows the simulated fading amplitude according to the model given by Equation (38) for the fixed frequency $f = 0$ as a function of $(x, y)$ in the pseudo-color representation (on a linear scale) for $a = 0.6$. We have simulated a sum of $N = 100$ sinoids. The figures demonstrate that the fading changes less in x-direction than in y-direction.

To build a PAS model with scattering clusters corresponding to distinct main angles of incidence $\hat{\alpha}_i$ ($i = 1, \ldots, M$), we propose utilize a superposition

$$S_{\text{angle}}(\alpha) = \sum_{i=1}^{M} \gamma_i p(\alpha - \hat{\alpha}_i; a_i)$$

(34)

of rotated wrapped Cauchy distributions $p(\alpha - \hat{\alpha}_i; a_i)$ with different opening parameters $a_i$ and relative power factors $\gamma_i$.

For such a distribution, the Fourier coefficients are simply a superposition of the Fourier coefficients of the single terms with some phase factors due to the angles of incidence. The construction of the random generator is also straightforward.

V. Generalization of the Model

A. Space Variance at Transmitter and Receiver

Up to now, we have considered the space variance of channel only at the receiver site. The same formalism can be applied if space variance only at the transmitter site is involved. In that case, the angle $\alpha$ has to be interpreted as the angle of departure (AoD). To include space variance at both the transmitter (TX) and receiver (RX) site, a straightforward generalization of Equation (2) leads to the transfer function

$$H(f, x, x') = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f \tau}$$

(35)

$$\times \int_{-\pi}^{\pi} d\alpha' e^{j2\pi l_{\alpha'} \cdot x} g(\tau, \alpha, \alpha').$$

As depicted in Figure 7, $x$ is the receiver (RX) location, $x'$ is the transmitter (TX) location, $\alpha$ is the angle of arrival at the receiver, and $\alpha'$ is the angle of departure, and $g(\tau, \alpha, \alpha')$ is a family of random variables that describes the contribution of the fading channel corresponding to the angles $\alpha$ and $\alpha'$ and the delay $\tau$. The frequency-space-space ACF is given by

$$R(f, x, x') = \int_{-\infty}^{\infty} d\tau e^{-j2\pi f \tau}$$

(36)

$$\times \int_{-\pi}^{\pi} d\alpha' e^{j2\pi l_{\alpha'} \cdot x} S(\tau, \alpha, \alpha').$$

The time-variance due to a moving transmitter and receiver can be included into the formalism by looking at $R(\Delta f, d + v\Delta t, d' + v'\Delta t)$, where $d$ and $d'$ are the distances between the respective RX and TX antenna pairs, and $v$ and $v'$ are the RX and TX velocity vectors. Now $S(\tau, \alpha, \alpha')$ is a more general scattering function that includes the topography at both sites. For practical calculations, however, it will be reasonable to make some factorization assumptions and to separate the RX and TX statistics and the delay statistics according to

$$S(\tau, \alpha, \alpha') = S_{\text{Delay}}(\tau) S_{\text{AoA}}(\alpha) S_{\text{AoD}}(\alpha').$$

(37)

Reasonable model assumptions about the three factors have to be made. For example, the delay profile $S_{\text{Delay}}(\tau)$ can be modeled by an exponential distribution (14) or by a superposition of such exponentials. For a downlink scenario, the scattering environment is richer at the receiver site than at the transmitter. In that case, $S_{\text{AoA}}(\alpha)$ may be described by Equation (34) with several distinct angles of arrival $\hat{\alpha}_i$ and opening angles related to parameters $a_i$. At the base station transmitter, the angle of departure is usually quite small, and one may describe the scenario by only one wrapped Cauchy distribution with a value of $a$ that is close to one.

For the simulation model, Equation (23) has to be extended to

$$H_{\text{model}}(f, x, x') = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j\theta_n} e^{-j2\pi f \tau_n} e^{j2\pi l_{\alpha_n}} x'$$

(38)

with $l_{\alpha_n} = \lambda_n^{-1} (\cos \alpha_n', \sin \alpha_n')^T$. 

315
Now consider a MIMO (multiple input – multiple output) system with $N_{TX}$ transmit antennas and $N_{RX}$ receive antennas. For simplicity, we assume a moving receiver only, but a static transmitter. In that case the time-variant model transfer function $H_{ik}(f,t)$ corresponding to RX antenna $i$ at relative location $\mathbf{x}_i = (x_i, y_i)^T$ and TX antenna $k$ at location $\mathbf{x}_k' = (x_k', y_k')^T$ is given by the following generalization of Equation (39):

$$H_{ik}(f,t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j \zeta_{i,k,n}} e^{j 2\pi \nu_n t} e^{-j 2\pi f t_n} \tag{39}$$

Here we have introduced the doubly space-dependent phase

$$\zeta_{i,k,n} = \theta_n + \frac{2\pi}{\lambda_0} (x_i \cos \alpha_n + y_i \sin \alpha_n + x_k' \cos \alpha_n' + y_k' \sin \alpha_n'). \tag{40}$$

B. External Doppler Scattering

An additional source of time-variance may occur due to moving scatterers between transmitter and receiver. We shall call this extrinsic Doppler scattering – in contrast to the intrinsic Doppler effect that is due to the movement of RX or TX antenna locations and that is only a scaled space variance. Examples for extrinsic Doppler scattering are ionospheric scattering or moving vehicles or persons between transmitter and receiver. We write $\mu$ for the variable of this additional external Doppler broadening and obtain a frequency-time-space-space transfer function given by

$$H(f, \mathbf{x}, \mathbf{x}') = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt_n \int_{-\infty}^{\infty} d\mu e^{j 2\pi \mu t} \int_{-\pi}^{\pi} d\alpha e^{j 2\pi \alpha_0} \times \int_{-\pi}^{\pi} d\alpha' e^{j 2\pi \alpha_0'} \times g(\tau, \mu, \alpha, \alpha'). \tag{41}$$

The corresponding ACF is given by

$$R(f, t, \mathbf{x}, \mathbf{x}') = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt_n \int_{-\infty}^{\infty} d\mu e^{j 2\pi \mu t} \int_{-\pi}^{\pi} d\alpha e^{j 2\pi \alpha_0} \times \int_{-\pi}^{\pi} d\alpha' e^{j 2\pi \alpha_0'} \times S(\tau, \mu, \alpha, \alpha'). \tag{42}$$

It may be a reasonable approximation to split up the comprehensive scattering function $S(\tau, \mu, \alpha, \alpha')$ into four factors:

$$S(\tau, \mu, \alpha, \alpha') = S_{\text{Delay}}(\tau) S_{\text{Doppler}}(\mu) S_{\text{AoA}}(\alpha) S_{\text{AoD}}(\alpha') \tag{43}$$

In contrast to the time variance caused by the vehicle motion of transmitter or receiver, it may here be reasonable to separate the time variance due to external Doppler broadening $S_{\text{Doppler}}(\mu)$. Furthermore, it may be reasonable in many scenarios to separate the transmitter and receiver azimuth statistics given by $S_{\text{AoA}}(\alpha)$ and $S_{\text{AoD}}(\alpha')$. However, all such assumptions have to be justified by physical reasoning.

VI. DISCUSSION AND CONCLUSIONS

We have described a model for the appropriate simulation of a quite general mobile radio channel. The model includes frequency selectivity and joint space-time variance. It takes into account that the time variance due to the motion of the receiver or transmitter has its source in the space dependency of the electromagnetic field and must therefore not be modeled independently. The only scenario where independent time variance can occur is the case of external Doppler broadening due to moving scatterers between transmitter and receiver.

The model has been stated quite generally to include many possible scenarios. A typical example we had in mind is a downlink scenario using Alamouti transmit diversity [11] with two antennas and a mobile receiver with two or more antennas. For computer simulations as well as for analytical considerations, we found it very convenient to model the angular distribution (PAS) of the incoming and outgoing wave by a wrapped Cauchy distribution or as a superposition of a few of such distributions. For such a model, random numbers can very easily be generated to perform computer simulations of the channel. Furthermore, the PAS has a very simple Fourier series which leads to a simple Fourier series for the spatial autocorrelation function. Eigenvalues of correlation matrices which are needed to calculate pair error probabilities and channel capacities can thus easily be obtained.

Besides its benefits, one should always keep in mind the limits of any model. For the model under consideration, we have assumed that the scatterers are ‘far away’ so that the angles and the delays do not change significantly during transmission. For a broadcasting or a classical mobile radio scenario, this assumption is less critical than for a radio link inside a building. However, the notions of a PAS and a Doppler spectrum and a power delay spectrum are not properly defined in the common way if the environment is rapidly changing. It should be a matter of serious discussion if it is helpful and if it leads to deeper insight to includes such things into the model. Another assumption that can be questioned is the uncorrelated scattering (US) model as stated by Equation (3). Even though this is obviously an idealization of the physical reality, it is well established since the famous paper by Bello [1]. Nearly all investigations are based on it. The assumption is crucial for building the model because it is equivalent to (second order) translation invariance in frequency and in space. For a moving receiver, space translation invariance becomes equivalent to the famous wide-sense stationarity (WSS). Without the assumption (3), again the PAS and the Doppler spectrum and the delay spectrum would not be defined. There are recent attempts [12] to do without this assumption. It would be helpful to get further
results that allow to estimate the impact of that assumption. We must always keep in mind the essential question: Does a more complicated model lead to more realistic results?

REFERENCES


