Growth Theory for a Monetary Production Economy Integrating the Treatise and the General Theory

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Integrating the Treatise and the General Theory

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Summary:
Solows' growth theory addresses two questions: (a) how does the economy grow into a stationary state equilibrium and (b) what will happen, if the growth of external factors induces a growth process. Both questions can be addressed in a Keynesian context too – although they are answered rather differently, as here demand and not some factor endowment is the restrictive factor. A model is presented, which integrates the multiplier of the General Theory and the reasoning of the Treatise to obtain a growth theory.

Zusammenfassung:
Die (alte) neoklassische Wachstumstheorie stellt zwei Fragen: (a) wie wächst die Ökonomie in ein stationäres Gleichgewicht und (b) was passiert, wenn Wachstum durch exogene Faktoren induziert wird. Diese Fragen lassen sich auch keynesianisch beantworten, wenn auch die Ergebnisse ganz andere sind, weil der Wachstumstreiber die Nachfrage ist und der Faktoreinsatz sich anpassen kann. Dies wird in einem Modell dargestellt, das Keynes' Ansätze aus der Treatise on Money und der General Theory integriert.

Subject(s): Growth Theory, Keynes, Treatise, Multiplier

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Bastard Keynesianism merged Keynes and neoclassical ideas by supposing that Keynes was valid only in the short run, in which prices (especially: wages) were sticky, while in the long run the price mechanism would ensure the attainment of the neoclassical (full-employment) equilibrium. Thus demand depended on current income and therefore the multiplier equation was of interest only if current income, due to some shock, departed from equilibrium income.

This reasoning is in some contrast to the use of the multiplier in the General Theory. Here the multiplier equation describes a state, in which income (and thus: supply) already has adjusted to demand. It is more or less a somewhat extended version of chapter 12.2 of the Treatise, describing the situation after the banana production has adjusted to the change in the savings rate, without describing the adjustment path.

So the General Theory claims to be a theory of the long run (basically of a stationary state), not of short run fluctuations. (It is not titled “A General Theory of short run Disequilibria” after all.)

The very point is that Keynes changes the causality of “classical” economics: It is not the productive capacity (given by the basic factor endowment of the economy) which determines possible output and to which demand (instantly or eventually) adjusts, but is is (effective) demand to which the capacity of the economy will eventually adjust – by (in case of labour) varying the amount of unemployed factors and by (in the case of produced input goods, aka capital) producing additional inputs.

Thus the multiplier equation describes stationary state equilibria only, which is not a very satisfying situation for a growth-theory, as neither the adjustment process is described nor is there any hint of how

---

2 I would like to thank Martin Ehret, Norbert Olah, Frank Raulf, and Alexander Troll for their comments and critique on an early version of this paper.
to explain a permanent growth process. In this paper I will try to mend
this problem by bringing together the Treatise – which explicitly deals
with disparities in supply and demand and therefore with the short run

1 Preliminary Considerations

There are two questions which have to be addressed in advance. Is
there any connection between growth and a stationary state? This is
answered in the affirmative, taking a look at Solows' growth theory: It
first describes the attainment of a stationary state and then asks, what
will happen, if that stationary state, due to external forces, starts to
change in time.

The second question follows from the interrelation of demand and
supply. If capacities have to change in order to adjust to demand the
input coefficients will have to be discussed. It is argued here, that in
the logic of a monetary theory of production, in which the rate of profit
is determined on the credit- and asset markets, it is quite natural to
assume fixed coefficients, as both input quantities can adjust.

1.1 A Glimpse at Solow

It is helpful to start with the consideration what the workhorse of
the neoclassical growth-theory actually does. Solows' growth theory
actually consists of two distinct parts. Solow starts at full-employment
(or “a given level of employment” Solow (1970)) but at some arbitrary
capital stock. The theory then asks what will happen to the capital
stock (and income), if the actual capital stock differs from the
stationary state capital stock – which is determined by the average
savings rate. If the actual capital stock is too low, savings will exceed
depreciation, raising capital and thereby income (and real wages),
while if the reverse is true, capital, income, and wages will adjust
downward toward a stationary state, determined by the rate of savings,
the level of employment (assumed to be fixed) and the production
function, describing the marginal productivity of capital for the given
amount of labour.

So basically in the first part of Solows' growth theory there is no
growth theory at all: it describes the adjustment path of an economy
which starts away from its long run equilibrium which eventually will
lead to its stationary state equilibrium in which by definition no further growth occurs.

Only in the further parts factors are identified, which may explain permanent growth: population growth (which, thanks to a given rate of employment, implies rising employment) and technological progress, and their influence on income, wages, and per capita income are discussed.

This paper will in principle follow Solows' procedure: at first the adjustment path to a stationary state equilibrium will be discussed – although this equilibrium of course is a Keynesian one – characterised by the multiplier equation and not by supply conditions.

The point here is simple: Supply is restricted by demand, and therefore in the long run adjusts to it. How how this adjustment exactly works will be discussed. I may mention already here, that this implies taking the multiplier equation as phenomenon of the long run and not of the short run.

The second part will only briefly identify external factors, which might induce growth.

The (no longer so) “new” growth theory (Lucas, Romer) tries to endogenize some of Solows' exogenous factors – be it the labour force, as additions to human capital increase the “efficiency units” of labour and the investment in human capital can be explained endogenously. Or be it technological progress, which, if it can be privatized thanks to patent laws, can also be endogenized. I won't have a go on this in this paper.

1.2 On Input-coefficients

One of the criticisms of Harrods' paper is that his result of a knife-edge growth path is caused by the assumption of a fixed capital coefficient. It is however this assumption which is the most Keynesian element of his model. The reason is as follows.

Consider the production price model. Here an individual technology is defined by its input-coefficients (with \( k \), \( l \) as the input-coefficients for the capital good and labour respectively), while the technology set is the set of all currently known technologies.

So suppose there are three known production processes, each described by the inputs required to produce one unit of output.
\( T_1: x = \min\{(1/0,2)K; (1/0,5)L\} \)
\( T_2: x = \min\{(1/0,3)K; (1/0,3)L\} \)
\( T_3: x = \min\{(1/0,4)K; (1/0,25)L\} \)

with \( T_i \) as technology \( i \), \( x \) as output, and \( K \) and \( L \) as the amount of capital and labour inputs respectively.

The technology set \( \mathcal{T} \) then would be described as
\[ \mathcal{T} = \{T_1, T_2, T_3\} \]

With technological progress meaning that some new technology is added to the set, which at least for some combination of factor prices is economically efficient.

The enveloping of the factor price frontiers of these three technologies describes the economically efficient production possibilities – or the factor price frontier for the whole technology set / the whole economy (Graph 1).

Graph 1: Factor Price Frontier
Let the economy be in a position characterized by a low wage rate and a high rate of return (say $w/p = 1$; $(1+r) = 2.5$). Then technology $T_1$, drawn with the red fpf, will be superior.

All three technologies could be used to produce the same amount of output. Their isoquants are depicted in graph 2, with the isoquants of $T_1$, $T_2$, and $T_3$ drawn in red, green, and blue respectively.

Graph 2: Isoquants

As Joan Robinson remarked, there is no fundamental difference between a model with a limitational and a model with a substitutional production function. Given enough (well, an infinite amount of) feasible technologies, you can always approximate the isoquant of a substitutional production function, which in Graph 2 is drawn in grey.

So, basically, a substitutional production function such as Cobb-Douglas describes the properties of the technology set $\mathcal{T}$, while a limitational production function describes the properties of a single technology $T_i$. 
Now, the basic difference between classical and Keynesian economics on the one hand and neoclassical economics on the other is their view of factor endowment.

For neoclassical economics the supply of (or rather: The supply function for) capital (savings) and labour is given, and all resources are used, so the economy is resource-constrained.

For the economy to grow, given the state of technological knowledge, factor endowment has to grow, so that, assuming a Cobb-Douglas PF:

\[ g_Y = \alpha \cdot g_K + (1 - \alpha) \cdot g_L \]

In classical economics additional supply of labour comes forward at the reproduction wage rate, so that both the growth rate of output and the growth rate of the labour force are determined by the growth rate of the capital stock:

\[ g_Y = g_L = g_K \]

In Keynesian economics the economy is demand constrained: demand determines equilibrium output and that in turn determines factor utilization: capital goods are produced as needed, while employment can expand as there are involuntarily unemployed people, which can be absorbed into the labour force. So:

\[ g_L = g_K = g_Y \]

This situation is depicted in Graph 3. Let the economy start at an employment level of .5 and an output level of 1. Then in all three paradigms the factor prices can prevail which render the red technology superior (say w/p = 1; (1+r) = 2.5).

But what will happen, if there is growth, say if the level of output doubles? Neither in classical nor in Keynesian theory there is any reason for factor prices to change, as the quantities of inputs can adjust. So in both cases employment and capital inputs will double, the input coefficients and factor prices will remain unaffected. The economy will expand along the dashed line, retaining the red technology.

In the Solow model however production is constrained by the given amount of labour, here 0.5. So the only way to reach a higher isoquant is by switching from the red to the blue technology producing with the same amount of labour, but with a higher capital/labour ratio.
(continuous line). (As can be seen in the fpf diagram, for this technology to become economically efficient, the real wage rate has to increase and the rate of return has to decrease.)

Graph 3: Expansion Paths

Measurement of capital. Capital in any sector is the (scalar) product of the vector of input goods and the price vector, and the total stock of capital is the sum of the capital of the individual sectors. The problem here is, that with a change in technology both the input-coefficients and the relative prices change. This implies, that “capital” as a measure of the quantity of capital goods is a problematic concept. But on the other hand, the same is true for the price level, income or employment – it stems from the fact, that heterogeneous units have to be aggregated using their prices and that generally neither the relative quantities remain constant between the periods to be compared (Bliss (1975)). So its a general problem of aggregation and if one wants to avoid it, one would have to avoid macro-economics all together.

Re-switching should not be a problem in neoclassical growth theory, as here the level of output grows. As one input (labour) is
assumed to be fixed, output can only increase, if one or more of the other input goods increase, because otherwise the process at the starting point would have not been technically efficient and therefore not economically efficient. Obviously the restriction on labour inputs renders re-switching to an earlier technology at a higher level of output infeasible.

Both considerations are not important for the classical and the Keynesian variety of growth-theory, as here the inputs adjust, so that relative prices remain unchanged.

There are however a few simplifications, which merit mentioning:

Solow models growth for a given participation rate – while during the growth process capital intensity and therefore labour productivity and wages increase. Actually employment should respond to a change in the wage rate, an effect which Solow doesn't model. So take the given participation rate as a simplification which allows to concentrate on the underlying logic of the neoclassical argument.

The classical variant would be modified if you think along the lines of Ricardos' argument: increasing employment increases the share of rental income at the expense of profits. The relative prices of food-stuffs and raw materials will change thus inducing a change in technology.

For the Keynesian version it can not be ruled out that the real rate of interest might change during the growth process, e.g. if in the monetary Keynesian case land, the surface of which is fixed, is a relevant portfolio alternative to nominal balances.

Nevertheless those three simplifications – given amount of labour, given real wage, and given rate of profit – serve well to stress the differences between the three paradigms and their implications.

2 Growth as Adjustment to a (new) Stationary State

2.1 The Stationary State

Following the reasoning in section 1.1 I will start with describing the stationary state equilibrium of the economy. The stationary state will be reached, if the supply of goods and services is equal to demand out of factor income, so demand determines supply. A closed economy is considered, so that demand is equal to the demand of the three
sectors: consumption demand of households, investment demand of firms and government spending:
\[ Y^D = C + I + G \]

with the equilibrium condition
\[ Y^D = Y = Y^S. \]

Consumption Demand

Consumption depends on disposable income. As income in the stationary state is, well, stationary, it is of no consequence, which consumption hypotheses exactly you choose, because previous income is equal to current income is equal to permanent income.

I therefore use the most simple form of:
\[ C = C_0 + c \cdot Y^V \tag{1} \]

while later on, in the simulations of the adjustment path, a variant of income persistence will be assumed. Here \( C \) is consumption demand, \( C_0 \) is autonomous consumption, \( c \) the marginal rate of consumption and \( Y^V \) is disposable income, which in turn is defined as income minus taxes:
\[ Y^V = Y \cdot (1 - t) \tag{2} \]

with \( t \) as the marginal tax rate.

\( c \) is influenced by income distribution as the average marginal rate of consumption is the weighted average of consumption out of wages (\( W \)) and profits (\( Q \)):
\[ c = c_w \cdot (W/Y) + c_0 \cdot (1 - (W/Y)) \tag{3} \]

As empirically \( c_w > c_0 \), a higher profit share will imply a lower marginal rate of consumption.

Investment Demand

As in a stationary state the capital stock doesn't change, investment demand (\( I \)) has to be equal to depreciation (\( D \)), which in turn is equal to the capital stock \( K \) times the rate of depreciation \( d \):
\[ I = D = d \cdot K^* \tag{4} \]
The equilibrium capital stock \((K^*)\) in turn is equal to the capital stock required to produce equilibrium output, or income times the capital coefficient.

\[
K^* = Y^* \cdot k \tag{5}
\]

so in stationary state investment demand will be (4) and (5)

\[
I = d \cdot Y^* \cdot k \tag{6}
\]

Later on investment will include net investment, so that the capital stock adjusts to changes in income, but for the determination of the stationary state equilibrium (6) is sufficient.

**Government Sector**

The government sector collects taxes \(T\) with

\[
T = t \cdot Y \tag{7}
\]

and buys goods and services \(G\). For short run analysis \(G\) usually is taken as autonomous, so that changes in tax income affect the budget balance only. Thus a marginal savings rate of 1 is postulated for the government sector, and in consequence the tax multiplier is the lower, the higher the tax rate.

I here assume, that the state spends its tax revenue, so that:

\[
G = T = t \cdot Y \tag{8}
\]

in principle you further could assume some permanent primary deficit, which would lead to a stable debt ratio. I don't do that here, as such a policy would turn out to lead to a lower level of stationary state income, because:

(a) You would have to run a primary surplus thus redistributing income from workers to wealth owners, thereby lowering the marginal rate of consumption because of (3). (As the rate of growth is zero, the primary surplus required to hold the debt ratio constant is equal to the real rate of interest times the debt share.)

(b) A higher debt share would mean a higher (real) rate of interest as the share of nominal assets in the portfolios would have to rise (Betz 2015).

---

3 For this argument see Betz (2012)
The Multiplier

Summing up, demand in stationary state is described by equations (1), (2), (6), and (8):

\[ Y^D = C_0 + c \cdot Y \cdot (1 - t) + d \cdot Y \cdot k + t \cdot Y \quad (9) \]

which, for \( Y = Y^S = Y^D = Y^* \), gives rise to the multiplier equation:

\[ Y = \frac{1}{1 - c - d \cdot k - t \cdot (1 - c)} \cdot C_o \quad (10) \]

with \( 1 > c \cdot (1 - t) - d \cdot k - t > 0 \) as the stability condition, which basically means that people can't plan to (permanently) spend more than they earn.

As can be seen, the multiplier values for \( c, k, d, \) and \( t \) will be positive, with the multiplier for the tax rate being:

\[ \frac{\delta Y}{\delta t} = \frac{1 - c}{(1 - c - d \cdot k - t \cdot (1 - c))^2} \cdot C_o > 0 \quad (11) \]

which of course is a consequence of the Haavelmo Theorem, as the state increases its tax income in order to raise government spending.

Supply

Output (and thus real income) is equal to the capital stock times capital productivity:

\[ Y = (1/k) \cdot K \]

Employment adjusts to \( K \) with

\[ L = (l/k) \cdot K \]
as there is no assumption of full-employment.

2.2 Modelling the Adjustment Path

In what follows the adjustment path to a new stationary state will be described under different assumptions. In 2.2.1 stable prices are assumed. (Prices are measured in wage units, i.e. \( w \) is assumed to be constant.) This assumption however is an improper assumption. It will therefore be lifted in 2.2.2, where – in line with the Treatise – deviations of supply and demand lead to shifts in the price level, which redistribute income. This, according to (3) will influence the marginal rate of consumption. Chapter 2.2.3 discusses a change in the tax rate
and Chapter 2.2.4 will model the influence of changes in the central bank interest rate on fluctuations of the price level and the adjustment path.

Consumption is modelled according to a simple habit persistence hypotheses with

\[ C = C_0 + c_0 \cdot Y_0^{FV} + c_{-1} \cdot Y_{-1}^{FV} \quad (1') \]

with 0 and -1 indicating the present and the previous period.

\( Y^F \) is factor income and therefore is equal to supply, which is determined by the capital stock of the respective period:

\[ Y^F = (1/k) \cdot K \quad (5') \]

In the first two simulations, the tax rate is assumed to be zero, so that there \( Y \) is equal to \( Y^F \).

The capital stock adjusts to demand: investment decisions reflect the discrepancy between the actual capital stock and the capital stock, that would have been required for the output to meet demand.

For ease of computation I assume these plans to be backward looking: Investment demand today reacts to the capacity gap\(^4\) experienced in the previous period, so that

\[ I_t = d \cdot K_{t-1} + \kappa (K^*_{t-1} - K_{t-1}) \]

or, as \( K^*_{t-1} \) is \( k \) times demand:

\[ I_t = d \cdot K_{t-1} + \kappa \cdot (k \cdot Y^D_{t-1} - K_{t-1}) \quad (12) \]

the first term on the right hand side is replacement demand, where as the second term describes planned capacity changes. \( \kappa \) allows for these capacity adjustments to be planned gradually. \( \kappa = .5 \) for instance would mean, that investors plan to close half of the capacity gap experienced in the previous period. In case of overcapacity the second term will become negative (so that the capital stock declines). \( I_t \) however is bounded from below as zero is the lower limit for every sort of demand.

It is quite reasonable to expect the pace of adjustment to be interest elastic. This possibility will be addressed in part 2.2.4.

\( ^4 \) As profitable investment is constrained by demand inputs become a function of equilibrium output. Thus in demand-speech it makes sense to talk of a capacity gap (as opposed to an output gap in supply speech) as an indicator of the capacity not yet in place to produce equilibrium output.
2.2.1 A First Approximation

The first model refers to a closed economy without taxes and government spending.

Factor income ($Y^F_t$) is described as
\[ Y^F_t = \left( \frac{1}{k} \right) \cdot K_t \]
with $k = 2.5$.

Consumption demand is habit persistent with the income of the previous period influencing current demand:
\[ C = C_0 + c_0 \cdot Y^F_0 + c_{-1} \cdot Y^F_{-1} \tag{1'} \]

The marginal rate of consumption is equal for both periods ($c_0 = c_{-1}$).
According to (3) it depends on income distribution:
\[ c = c_w \cdot \left( \frac{W}{Y} \right) + c_q \cdot \left( 1 - \frac{W}{Y} \right) \tag{3} \]
I assume $c_w$ to be .4, $c_q$ to be .2 for each period, and $W/Y$ to be equal to .5, so that $c_0 = c_{-1} = .3$.

Investment demand is a function of the capacity gap according to (12):
\[ I_t = d \cdot K_{t-1} + \kappa \cdot \left( k \cdot Y^D_{t-1} - K_{t-1} \right) \tag{12} \]
with $d = 0.1$ and the rate of capacity adjustment $\kappa = .3$

The starting values are 1000 for $K$ and 100 for $C_0$ so that $Y^* = 500$.

In period 0 the system is shocked: $C_0$ increases permanently to 150.

<table>
<thead>
<tr>
<th>Simulation 2.2.1$^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Values</td>
</tr>
<tr>
<td>$C_0$</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

5 As the rate of depreciation is assumed to be 10%, the actual capital coefficient is 0.2, thus the system is productive. In a production price model this would have to be modelled as joint-production.

6 The spreadsheet with the simulations is provided on the same web site as the paper, so that all simulations can be re-run with different parameters. (The file is in OpenOffice format *.ods)
Graph 4 shows the reaction in the income-expenditure diagram.

Demand takes off relative to supply, so that a surplus demand is generated (the difference between the blue equilibrium locus and the orange demand curve) which at first increases, as the surplus demand invokes additional investment demand, and eventually peters out, as the capacity adjusts to equilibrium demand.

Graph 4: Adjustment Path of Model 2.1

Accordingly the growth rate at first accelerates and then declines as the economy approaches its new stationary state (graph 5).

While giving a good first illustration of the basic concept, the simulation however is flawed in one respect: Although output is not sufficient to meet demand, it is assumed that all investment demand can be met, so that capacity increases by the amount of planned net investment. As Keynes demonstrated in his Treatise on Money, the mechanism which will equate supply and demand will be rising prices so that investment can crowd out consumption demand. This process will be included in the model in the following part.
2.2.2 Widows Cruse

In the Treatise, Keynes distinguishes equilibrium profits (Q*) - which he defines as the level of profits at which the entrepreneur would plan neither to expand nor to contract his production and unexpected windfall profits (Qu), which emerge, when demand outstrips supply. (If supply outstrips demand, windfall losses will occur: Qu < 0). Qu is equal to the difference between investment and planed savings:

$$Qu = I - S$$ or, alternatively

$$Qu = Y^D - Y^F$$

They occur, because surplus demand raises prices above factor costs by a factor $pg$, with

$$pg = \frac{I - S}{Y^F} = \frac{Y^D - Y^F}{Y^F} = \frac{Qu}{Y^F} \quad (13)$$

The reason for the Treatise to discuss this was Keynes' intention to provide an alternative for the Quantity Theory of Money: According to the Treatise, the price level is not determined by the quantity of money,
but by contracted factor costs (the wage unit of the GT) and by the relationship of aggregate supply and demand.\(^7\)

This implies, that an increase in investment redistributes income, thus reducing the purchasing power of factor income, and thereby crowding out consumption to make room for the additional investment demand. In contemporary discussion this process was known as “forced saving” (see for example Hayek (1932)).\(^8\)

Demand is defined as

\[
Y^D = C + I
\]

with

\[
C_t = C_0 + c_{-1} \cdot Y_{t-1}^F + c \cdot Y^F
\]

As factor income is redistributed by pg, income shares will change and thus will the average marginal rate of consumption:

\[
c_t = c_w \cdot (1 - (Q/Y) - pg_t) + c_Q \cdot ((Q/Y) + pg_t)
\]

Assuming that investment demand is defined in real terms (it is a function of the capacity gap after all) the increase in the price level will have to be sufficient to crowd out consumption and make room for investment.

Thus:

\[
\frac{C}{1 + pg} = Y^F - I <= (1 + pg) = \frac{C}{Y^F - I}
\]  \hspace{1cm} (14)

Investment enters the demand equation of the current period and adds to capacity in the following period, so that

\[
Y^F_t = k \cdot (K_{t-1} + I_t)
\]

---

7 Ironically, this implies that Keynes postulates fixed prices (measured in the wage units of the GT) for the long run and variable prices for the short run (as long as capacities have not adjusted). Thus the GT is concerned with the long run: When the multiplier has run its course \(Y^S = Y^F\) has adjusted to \(Y^D\), while the short run is analysed in the Treatise. This is the opposite of what textbook “Keynesian” theory assumes.

8 This is a point which Harrod overlooks, when he argues, that variations of the savings rate aren't sufficient to render his knife edge blunt (Harrod (1939), p 26) : If planned savings don't vary enough, unplanned savings will make up the rest, thus allowing capacities to grow faster. That he does not consider this mechanism is a bit surprising, because in other parts of his article he refers quite extensively to the Treatise.
Graph 6: Model 2.2. Investment crowds out Consumption
while $I$ depends on depreciation and the capacity gap of the previous period as stated in (12). Again, as noted above, investment demand is taken to be defined in real terms.

The dynamics of a shock to autonomous consumption are illustrated in graph 6. (Co increases from 100 to 120 in period 0). Except for the shock to Co parameters are the same as in 2.2.1. (To avoid circularities, I use $p_{t-1}$ for the computation of $c_t$.)

Starting from the upper graph with the gap between output and demand: In period 1 consumption demand rises. Thus a surplus demand emerges, which induces additional investment demand. Because of this the capacity gap at first increases further. When the new investment starts to add to capacity, the capacity gap slowly closes, as the capital stock adjusts to demand.

Thus the growth rate accelerates in the first periods (as, in line with the accelerator model, the increase in consumption demand induces additional investment) and then slowly declines (as capacity adjustment peters out). Given the relatively low value of $\kappa$, this adjustment takes quite some time. Even after 30 periods the growth rate still is in the region of 1%.

$pg$, which equates supply at factor cost and demand at market prices, starts at a high level, increases further in the first few years and slowly vanishes, as the capacity gap closes.

The windfall profits associated with the increase of market prices over factor costs lead to a drop in the wage share, so that $W/Y$ varies with surplus demand:

\[
\frac{W}{Y} = \frac{W}{Y^F} \cdot \frac{1}{1 + pg} \quad (15)
\]

With the capacity gap closing, the wage share will eventually recover to reach its previous value.

It is to be expected that the initial amplitude of $pg$ and the wage share will be dampened, if one allows for changes in inventories which absorb some of the surplus demand. But then, one would aspect the adjustment process to be drawn out, as the inventories eventually will have to be replenished, leading to higher investment demand in later periods.
2.2.3 Monetary Policy

The problem with the adjustment process is, that the reduction of the wage share by way of inflation in the early stages of the process can easily induce a price-wage-price-spiral.

Going back to the Treatise, the price level \( P \) can be derived as follows: Let \( P \) be the price level, \( Q \) and \( W \) be the sum of nominal wages and profits, with \( Q = Q^* + Qu \) as defined above.

Then

\[
P \cdot Y^F = W + Q^* + Qu
\]

which Keynes rearranges to

\[
P = \frac{Q^* + W}{Y^F} + \frac{Qu}{Y^F} = p_Y + pg
\]

The price level is the sum of the price level in equilibrium (\( p_Y \)) and \( pg \), which is due to excess demand which has already been discussed in (13). What remains to be determined is the equilibrium price level \( p_Y \).

\[
p_Y \cdot Y^F = W + Q^* \quad (16)
\]

Express \( Q^* \) as a mark up (\( m \)) on wage costs (\( Q^* = m \cdot W \)) and divide both sides of the equation by \( Y^F \). Then, as the wage bill is nominal wages times employment (\( L \)) (16) becomes

\[
p_Y = w \cdot \frac{L}{Y^F} \cdot \left( 1 + m \right) \quad (16').
\]

\( L / Y^F \) is the input coefficient of labour (\( l \)), while \( m \) can be explained as the rate of return on capital (equal to the rate of depreciation plus the (equilibrium) rate of profit \( r \)) times capital intensity (\( k/l \)). Thus

\[
p_Y = w \cdot l \cdot \left( 1 + \frac{k}{l} \cdot (r + d) \right) \quad (16'')
\]

Given \( r \) and the technology set, \( l, k \) and \( d \) are determined, so that \( p_Y \) depends on the nominal wage rate only and is linear homogenous in \( w \).

---

9 Or, in terms of production price model:

\[
p = (I - A \cdot (1+r)) \cdot l \cdot w
\]

with \( p \) as the price vector, \( I \) the identity matrix, \( A \) the matrix of input coefficients and \( l \) the vector of labour coefficients. Again, given \( r \) and the technology set, \( A \) and \( l \) are determined, so that \( p \) is linear homogenous in the nominal wage rate. See also: Riese (1980).
Graph 7: Simulations for different Values of $\kappa$

- YD - YS

- Growth Rate

- PG

- Wage Share
In the long run therefore the rate of inflation has to be equal to the
growth of nominal wages (minus the growth rate of productivity, if
technological progress takes place and thus the technology set
changes).

Thus, if the nominal wage rate doesn't change, pg does not pose a
problem of inflation, as it will eventually return to zero, as the capacity
gap closes. It will however become critical, if a price level rise caused
by pg induces an increase in nominal wages, as workers try to fend off
a fall in the wage share (and in the purchasing power of their wages).

As Graph 7 shows, the initial increase of pg will react drastically to
the speed of capacity adjustment: The higher $\kappa$, the higher the initial
amplitude of pg. Now it is natural to assume that the (real) interest rate
of the central bank can influence $\kappa$ as it influences the market rate of
interest. If the market rate of interest is perceived to be high relative to
long run interest expectations, it makes sense to delay investment. So
by leaning against the wind the central bank can dampen the initial
changes of pg and the wage share and thus reduce the danger of a
price-wage-price spiral. The flip side of this of course is, that for lower
values of $\kappa$ it will take longer for the capacity gap to close. (Which in
turn means, that the growth rate will be lower initially and higher in
later periods. Thus employment will be lower through out the
adjustment process.)

The lag structure of the model involves a second order differential
equation: $K_t$ (and therefore $Y^F_t$) depends on investment demand in $t - 1$
which in turn depends on the capacity gap of period $t - 2$. For higher
values of $\kappa$ the model therefore starts to oscillate. For values of $\kappa$
greater than .7 those oscillations explode. These values are not
simulated, as the development of the series violates some natural
boundaries: Neither can gross-investment nor factor income become
negative nor can real investment exceed real output.

Further more the oscillations will be damped, if one allows
adjustments in inventories or some short-term deviation from
equilibrium input-coefficients (for instance by extra shifts).
2.2.4 Fiscal Policy

There are two aspects to fiscal policy.

The first is, that it too can contribute to the dampening of fluctuations. Anti-cyclical policy implies, that it runs surpluses in the boom, thereby increasing the planned saving of the economy and thus reducing Qu and therefore pg and vice versa in the slump. This, however, is nothing new, and it therefore is not simulated.\textsuperscript{10}

The other aspect is the size of the states-share. According to (7) an increase in the tax rate will finance higher government demand. It thus increases demand by taxing away income of which only a part would have been spent and spending all of it.

Graph 8 gives the simulation results for an increase in the income tax rate from 0 to 10%. The eventual increase in $Y_F$ exceeds the Haavelmo-Multiplier ($dY_F > dG$), because the additional output requires a higher capital stock, which in turn implies additional investment demand in equilibrium because of higher depreciation ($\Delta D = d \cdot \Delta K$).

Due to the additional government demand, a capacity gap emerges, which induces additional investment demand. Thus pg becomes positive, reducing the wage-share.

In consequence the economy starts to grow. With the capacity gap slowly closing, pg vanishes and the wage-share returns to its previous level. (Note, however, that we are talking about pre-tax wages. The share of net wages in factor income will be lower, as – with r given – wage income carries the tax-burden. Of course on the other hand the supply of public goods increases. So a lower wage-share does not imply that the formerly employed are worse off to the same degree as their net-wages are reduced.)

As in all previous simulations, the growth impact of the shock eventually peters out as the economy approaches its new stationary state.

\textsuperscript{10} The only aspect which differs from conventional theory is the functional chain: A budget surplus does not avoid over-employment and thus a wage-price-spiral. Instead it reduces pg and thus avoids a price-wage-spiral.
Graph 8: An Increase in the Tax Rate
2.2.5 Conclusions

All simulations discussed in Chapter 2.2 share a common feature: the growth effect of a one time shock to one of the parameters of the multiplier equation will eventually peter out, so no permanent growth process can be derived from there.

During the adjustment path the wage-share will deviate from its previous value as \( pg \) makes room for additional investment, thus increasing the profit-share. Contractions could easily be modelled. They would exhibit the opposite signs for \( pg \) and \( W/Y \).

If capacity adjusts quickly to demand the eventual equilibrium will be attained faster, but at high values of \( \kappa \) the system will start to oscillate.

Further shocks could easily be simulated: a shock to income distribution for instance would work as a shock to the marginal rate of consumption. Thus an increase in income inequality would lower equilibrium income and therefore induce negative growth rates (or lower growth rates, if some growth was induced by other factors at the same time).

In all simulations \( pg \) first increases and then declines (with the wage share as its mirror image) in response to an expansionary shock. Thus there are phases in which growth goes along with rising profits (growth seems to be profit-led) and phases, in which it goes along with rising (real) wages. As in all simulations phases of increasing windfall-profits are shorter than the periods of an increasing wage share it does not come as a surprise that empirical investigations find most growth-episodes to be “wage-led”.\(^{11}\)

\(^{11}\) See for instance Lavoie et alt. (2014)
3 Steady State Growth

Examining the multiplier equation (10), reproduced here for convenience:

\[ Y = \frac{1}{1 - c - d \cdot k - t \cdot (1 - c) \cdot C_o} \]

(10)

Continuous growth could be caused by either continuous changes of one or more parameters of the multiplier or by continuous growth in \( C_0 \).

Examining parameters of the multiplier first. As the simulations have shown, it can take quite some time for the economy to reach its new equilibrium if there is a one-time shock to one of the parameters. So continuous changes could explain quite extended growth spells. Nevertheless, there is a limit to this. First, because of the stability condition of

\[ 0 < 1 - c - d \cdot k - t \cdot (1 - c) < 1 \]

in which the left-hand inequality implies, that agents can not plan to spend permanently more than they earn, while the right-hand side implies, that (equilibrium) output can't be smaller than autonomous consumption, as you can't consume more than you produce.

Further more, there are some limits to the values which the individual parameters can attain.

\( c \) – can be influenced either by changes in (primary or secondary) income distribution with decreasing inequality associated with an increasing marginal rate of consumption. Here the limit would be defined by the highest marginal rate – which in turn should decrease, if the income dispersion shrinks, i.e. if the poor become relatively less poor. And / or it could be influenced by changing customs which make consumption relatively more attractive.

\( d \cdot k \) – both a higher rate of depreciation and a higher capital coefficient would raise equilibrium income. The limit here is, that the system has to be productive, producing more output then the input it consumes. Thus \( d \cdot k \) has to be lower than unity. Otherwise the economy would start to physically vanish from the surface of the earth.
Graph 9: Continuous Growth of Co
t – given r, only wage-income can be taxed. thus the (unrealistic) upper limit for increases in the tax-rate is not one, but W/Y. This still leaves room for extended economic phases such as the (expansive) phase of the emergence of the welfare state and the (contractionary) phase of its deconstruction by neo-liberal policies, but not for a continuous growth process.

So the main candidate for continuous growth is Co. If autonomous consumption grows at a continuous rate $g_{Co}$, then the economy will grow at the same rate.

$$g_Y = g_{Co}$$

Graph 9 describes the simulation for this case. Here again I start in a stationary-state equilibrium. From period 0 onwards Co starts to grow at a continuous rate of 2% per period. The growth-rate of output eventually adjusts to the same rate of growth, with the steadily increasing investment invoking permanent surplus demand thus causing extra profits and therefore a permanently lower wage wage-share than in the stationary state.

Thus, while an increase in inequality in the sense of a lower equilibrium wage share will be detrimental to growth (as it reduces the equilibrium income level towards which the economy growth) growth by itself reduces – at least with adaptive investment plans – the steady state share of the economy, because it again and again evokes the surplus demand, which creates extra profits. But note, that here not inequality is a pre-requisite for growth (because higher income-inequality increases the savings rate and therefore the amount of capital goods available as inputs) but that it is a consequence of growth, which it does neither induce nor accelerate.

These results however are extremely sensitive to the assumptions about investment plans. Would they react for example to the past trend instead of the size the past capacity gap, the adjustment to the new equilibrium growth rate would be faster, the extra profits would be higher in the first rounds and lower later on, so that the long-run

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12 Windfall profits can be taxed – but if spend they would re-emerge again, as surplus demand would remain unchanged. So they could only be used to finance a budget surplus, if the aim is to reduce Qu (and thus pg). Rents however may be taxed in order to affect income distribution.
deviation of the wage-share from its stationary state value would be smaller.

There are at least two possible influences on Co, which I see. The first obviously is population growth. If Co is a function of population, then the two growth rates should be linked.\textsuperscript{13} Then there may be a second influence: As Piketty reports, the size of housing wealth relative to GDP increases since the early seventies (while the capital-output ratio doesn't change that much). This might induce demand for repair (or replacement) which in the above model would show up in Co. Whether or not that can induce a long term growth-trend depends on whether the wealth to output ratio will increase continuously – at least in Pikettys' model, it does not and will eventually converge to some new equilibrium level.

Technological progress could eventually influence Co, if it is considered to consist of product innovation (although I find it hard to imagine a steady trend here). Otherwise it could only influence employment given the growth rate of output. (Possible effects on d and k have already be mentioned above.)

The same applies to changes in human capital – except if one assumes that a higher level of education has some influence on demand.

\textsuperscript{13} The problem with this is of course the general problem with exponential growth: there always comes a point, where the results become utter nonsense. Should population grow continuously at some positive rate, then one day the combined mass of its human inhabitants exceeds the mass of the whole planet. While in the case of economic growth the problem might be avoided, if resource intensity decreases while output grows (the BIP is a value and not a mass category after all) in the case of population growth you would have to assume, that the average mass of a human shrinks – which seems not to be the case ....
Conclusion

As has been shown, a growth theory for a Monetary Theory of production is possible and not, as Riese (1997) concludes, impossible for Keynesian theory – at least if one sticks to the questions that the (old) neoclassical growth theory addresses.

It differs fundamentally in its approach to these questions: growth has to be thought of as demand driven. And therefore its answers are radically different from neoclassical expectations too.
References


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